

Sequences

- Our first chapter of the course will involve recognizing patterns by using sequences. A **sequence** is an ordered group of numbers, symbols, or pictures in which each item (called a term) is determined by using a formula or rule. A sequence is written as a list of numbers in the form of $\{t_1, t_2, t_3, \dots\}$, where t_1 is the first term, t_2 is the second term, etc. Be careful to remember that terms begin at “1” and not at “0”.
- We will be looking at two different types of sequences in our first chapter: finite and infinite. A **finite sequence** is one that has an ending (a final or last term). The number of terms in a finite sequence can be counted and the sequence looks like this: $\{t_1, t_2, \dots, t_n\}$, where t_n is the last term or n^{th} term of the sequence.

○ Examples:

(a.)	$\{2, 5, 8, 11, 14\}$	(b.)	$\{3, 6, 12, 24, 48, 96\}$
	↑ ↑ ↑ ↑ ↑		↑ ↑ ↑ ↑ ↑ ↑
	$t_1 \ t_2 \ t_3 \ t_4 \ t_5$		$t_1 \ t_2 \ t_3 \ t_4 \ t_5 \ t_6$

- An **infinite sequence** is one that does not have an ending. The number of terms can not be counted and it looks like this: $\{t_1, t_2, t_3, \dots\}$, where the “...” tells us that it continues on to infinity.

○ Examples:

(a.)	$\{6, 10, 14, 18, \dots\}$	(b.)	$\{3, 3, 5, 9, 15, 23, \dots\}$
	↑ ↑ ↑ ↑		↑ ↑ ↑ ↑ ↑ ↑
	$t_1 \ t_2 \ t_3 \ t_4$		$t_1 \ t_2 \ t_3 \ t_4 \ t_5 \ t_6$

- Like linear and quadratic equations, we can identify the domain and range of a sequence. In any sequence, the **domain** refers to all the possible values of n (the x-values) and the **range** refers to all the possible values of t_n (the y-values).
- Example 1:
If we have the sequence $\{6, 9, 14, 21, 30, \dots\}$, we can think of it instead as a table of values:

n	1	2	3	4	5
t_n	6	9	14	21	30

The DOMAIN (n) is the values 1, 2, 3, 4, 5, ... or the positive integers. Therefore, $D = \{n \mid n \in \mathbb{N}\}$.

The RANGE (t_n) is the values 6, 9, 14, 21, 30, ... or the values of $n^2 + 5$. Therefore, $R = \{t_n \mid t_n = n^2 + 5, n \in \mathbb{N}\}$.

- A sequence can be represented in many different forms, such as a picture, using a graph, as a table of values or using algebra.

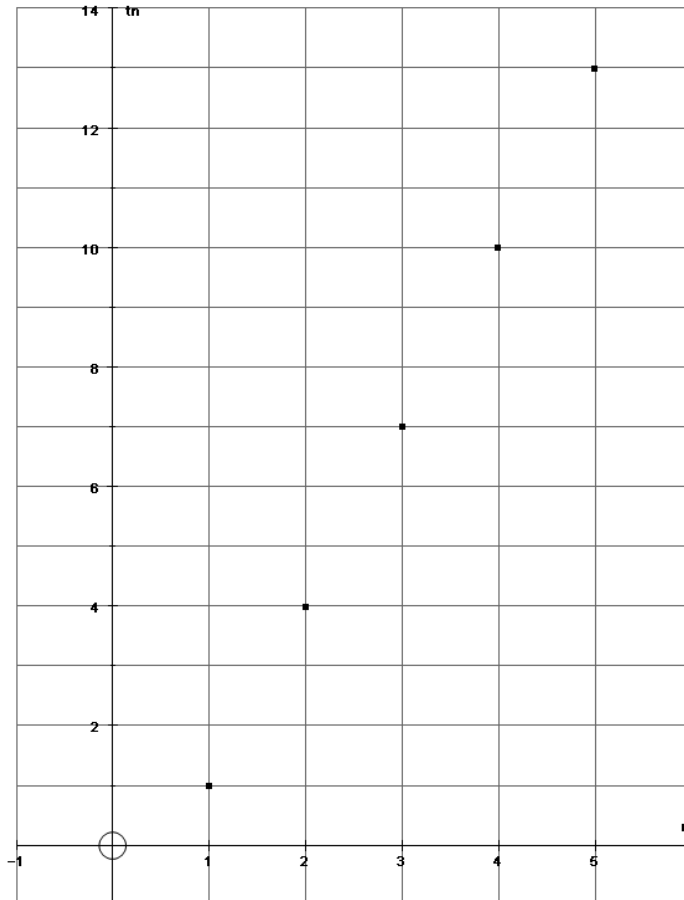
- Example 2:

Let's consider the sequence $\{1, 4, 7, 10, 13, \dots\}$, where $t_1 = 1$, $t_2 = 4$, $t_3 = 7$, etc.

(i.) Picture:



(ii.) Using a Graph:



(iii.) Table of Values:

n	1	2	3	4	5
t_n	1	4	7	10	13

(iv.) Using Algebra:

$$t_n = 3n - 2$$

- In Example 2, we see the same sequence represented in four different ways. The way we represent a sequence depends on the type of problem that we are working on. Usually, we will use the algebra method because it will help us to find the value for any term.

- Example 3:

A sequence is defined by the equation $t_n = 3n - 2$.

(a.) Find the values of t_7 and t_{200} .

(b.) Which term has a value of 172?

(a.) To find the value of t_7 ,

we let $n = 7$:

$$t_n = 3n - 2$$

$$t_7 = 3(7) - 2$$

$$t_7 = 21 - 2$$

$$t_7 = 19$$

To find the value of t_{200} ,

we let $n = 200$:

$$t_n = 3n - 2$$

$$t_{200} = 3(200) - 2$$

$$t_{200} = 600 - 2$$

$$t_{200} = 598$$

(b.) Let $t_n = 172$ and solve for n :

$$t_n = 3n - 2$$

$$172 = 3n - 2$$

$$174 = 3n$$

$$n = 58$$

Therefore, $t_{58} = 172$.