

Sequences

- This section of the course will involve recognizing patterns by using sequences. A **sequence** is an ordered group of numbers, symbols, or pictures in which each item (called a term) is determined by using a formula or rule. A sequence is written as a list of numbers in the form of $\{t_1, t_2, t_3, \dots\}$, where t_1 is the first term, t_2 is the second term, etc. Be careful to note that terms begin at “1” and not at “0”.
- We will look at two types of sequences in this chapter: finite and infinite. A **finite sequence** is one that eventually ends. The number of terms can be counted and it looks like this: $\{t_1, t_2, \dots, t_n\}$, where t_n is the last term or n^{th} term of the sequence.

○ Examples:

$$\begin{array}{cccccc}
 \text{(a.) } \{1, 3, 5, 7, 9\} & & \text{(b.) } \{2, 4, 8, 16, 32, 64\} \\
 \uparrow \uparrow \uparrow \uparrow \uparrow & & \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \\
 t_1 \ t_2 \ t_3 \ t_4 \ t_5 & & t_1 \ t_2 \ t_3 \ t_4 \ t_5 \ t_6
 \end{array}$$

- An **infinite sequence** is one that does not end. The number of terms can't be counted and it looks like this: $\{t_1, t_2, t_3, \dots\}$, where the ellipsis (“...”) tells us that it continues on to infinity.

○ Examples:

$$\begin{array}{cccccc}
 \text{(a.) } \{5, 10, 15, 20, \dots\} & & \text{(b.) } \{1, 2, 5, 14, 41, 122, \dots\} \\
 \uparrow \uparrow \uparrow \uparrow & & \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \\
 t_1 \ t_2 \ t_3 \ t_4 & & t_1 \ t_2 \ t_3 \ t_4 \ t_5 \ t_6
 \end{array}$$

- Just like linear and quadratic equations, we can identify the domain and range of a sequence. In any sequence, the **domain** refers to all the possible values of n (the x-values) and the **range** refers to all the possible values of t_n (the y-values).
- Example 1:
If we have the sequence $\{1, 4, 9, 16, 25, \dots\}$, we can think of it instead as a table of values:

n	1	2	3	4	5
t_n	1	4	9	16	25

The DOMAIN (n) is the values 1, 2, 3, 4, 5, ... or the positive integers. Therefore, $D = \{n \mid n \in \mathbb{N}\}$.

The RANGE (t_n) is the values 1, 4, 9, 16, 25, ... or the values of n^2 . Therefore, $R = \{t_n \mid t_n = n^2, n \in \mathbb{N}\}$.

- A sequence can be represented in many different forms, such as a picture, using algebra, using a graph or as a table of values.

- Example 2:

Let's consider the sequence $\{3, 5, 7, 9, 11, \dots\}$, where $t_1 = 3$, $t_2 = 5$, $t_3 = 7$, etc.

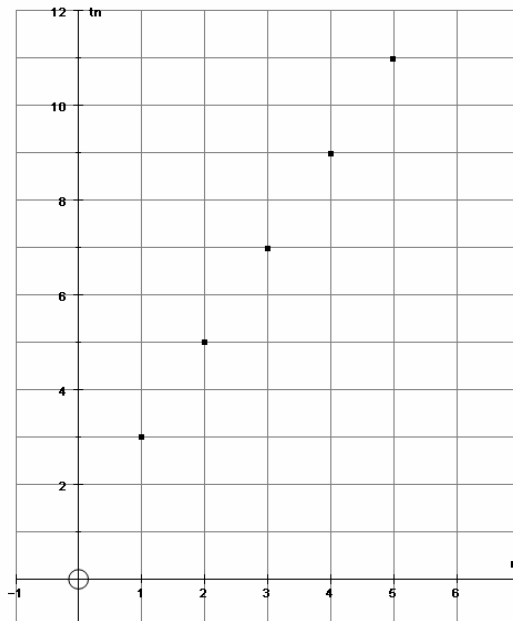
(i.) Picture:



(ii.) Using Algebra:

$$t_n = 2n + 1$$

(iii.) Using a Graph:



(iv.) Table of Values:

n	1	2	3	4	5
t_n	3	5	7	9	11

- In Example 2, we see the same sequence represented in four different ways. The way we represent a sequence depends on the problem that we're working on. Usually, we will use the algebra method because it will help us identify the value for any term.
- Example 3:
A sequence is defined by the equation $t_n = 2n + 1$.
(a.) Find the values of t_6 and t_{100} .
(b.) Which term has a value of 129?

<p>(a.) To find the value of t_6,</p> <p>we let $n = 6$:</p> $t_n = 2n + 1$ $t_6 = 2(6) + 1$ $t_6 = 12 + 1$ $t_6 = 13$	<p>To find the value of t_{100},</p> <p>we let $n = 100$:</p> $t_n = 2n + 1$ $t_{100} = 2(100) + 1$ $t_{100} = 200 + 1$ $t_{100} = 201$
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(b.) Let $t_n = 129$ and solve for n :

$$t_n = 2n + 1$$

$$129 = 2n + 1$$

$$128 = 2n$$

$$n = 64$$

Therefore, $t_{64} = 129$.