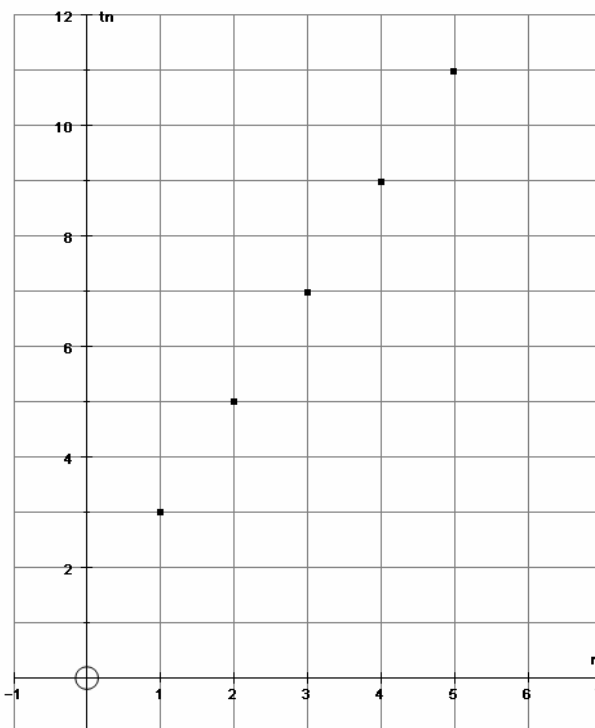


Arithmetic Sequences

- Let's look back at our sequence from the last lesson: $\{3, 5, 7, 9, 11, \dots\}$. When looking at each term, we notice that each consecutive term is increasing by two. This change in value is what we call a **common difference** (or "d"), because the difference between a term and the term before it is the same in each case. In general, the common difference for any two terms may be found by calculating $t_n - t_{n-1}$ or $t_{n+1} - t_n$. We can see this pattern below:

$$\begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & \\ \{3, & 5, & 7, & 9, & 11, & \dots\} \\ \vee & \vee & \vee & \vee & & \\ 2 & 2 & 2 & 2 & \leftarrow & \text{common difference} = 2 \end{array}$$

- This sequence is an example of what we call an **arithmetic sequence**. In an arithmetic sequence, the terms increase/decrease by a constant value (d), and the data looks like a linear relationship when we graph it.



- When looking at the graph of our sequence, we can see that it does look like it makes a straight line pattern (linear relationship). However, we **DO NOT** join the points together to make a straight line. Our data here is **discrete** because we don't have a $t_{1.876}$ term or a $t_{423.5}$ term. By looking at the graph, we can also find that $t_1 = 3$ and the common difference is two ($d = 2$). We can also find the formula for the sequence, which is $t_n = 2n + 1$.
- We should also note that the **common difference** of an arithmetic sequence is similar to the **slope** of a linear equation.
- We can find the formula for the sequence from the graph by using what we know about linear equations, but how can we find the formula by just using the sequence?

Finding the Equation of an Arithmetic Sequence

- An arithmetic sequence with n terms can be defined by the formula $t_n = t_1 + (n - 1)d$, where t_1 is the first term of the sequence and d is the common difference.
- Using our sequence from the previous page, we have:

$$t_n = t_1 + (n - 1)d$$

$$t_n = 3 + (n - 1)(2)$$

$$t_n = 3 + 2n - 2$$

$$t_n = 2n + 1$$

This formula is exactly the same as the one we found from using the graph.

- Example:
Find the formula for each arithmetic sequence given below:
(a.) $\{1, 5, 9, 13, 17, \dots\}$ (b.) $\{5, -2, -9, -16, -23, \dots\}$
(c.) $\{3.4, 3.6, 3.8, 4.0, 4.2, \dots\}$

$$(a.) \quad t_n = t_1 + (n - 1)d$$

$$t_n = 1 + (n - 1)(4)$$

$$t_n = 1 + 4n - 4$$

$$t_n = 4n - 3$$

$$(b.) \quad t_n = t_1 + (n - 1)d$$

$$t_n = 5 + (n - 1)(-7)$$

$$t_n = 5 - 7n + 7$$

$$t_n = -7n + 12$$

(c.) $t_n = t_1 + (n - 1)d$
 $t_n = 3.4 + (n - 1)(0.2)$
 $t_n = 3.4 + 0.2n - 0.2$
 $t_n = 0.2n + 3.2$