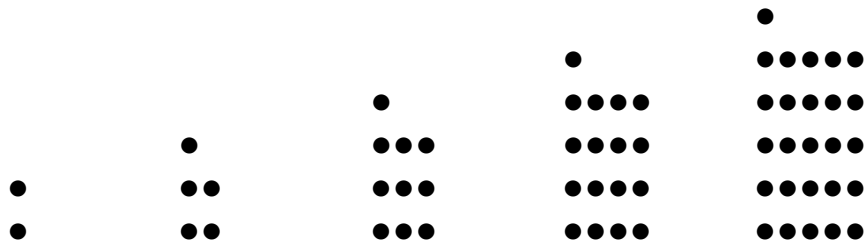


Quadratic Sequences

- As we have seen before, arithmetic sequences have a common difference between successive terms. The sequence $\{1, 4, 7, 10, 13, \dots\}$ is arithmetic because it has a common difference of 3. There are other types of sequences that are not arithmetic. We are going to look at another type of sequence called a **quadratic sequence** by working through an example.

Finding a Quadratic Pattern

- We are given the following diagram:



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

- The first step is to list the first five terms of the sequence that represent the number of dots plotted. The sequence looks like this: $\{2, 5, 10, 17, 26, \dots\}$.
- The second step is to check if the sequence generated is arithmetic.

Check:

$$t_2 - t_1 = 5 - 2 = 3 \qquad t_3 - t_2 = 10 - 5 = 5$$

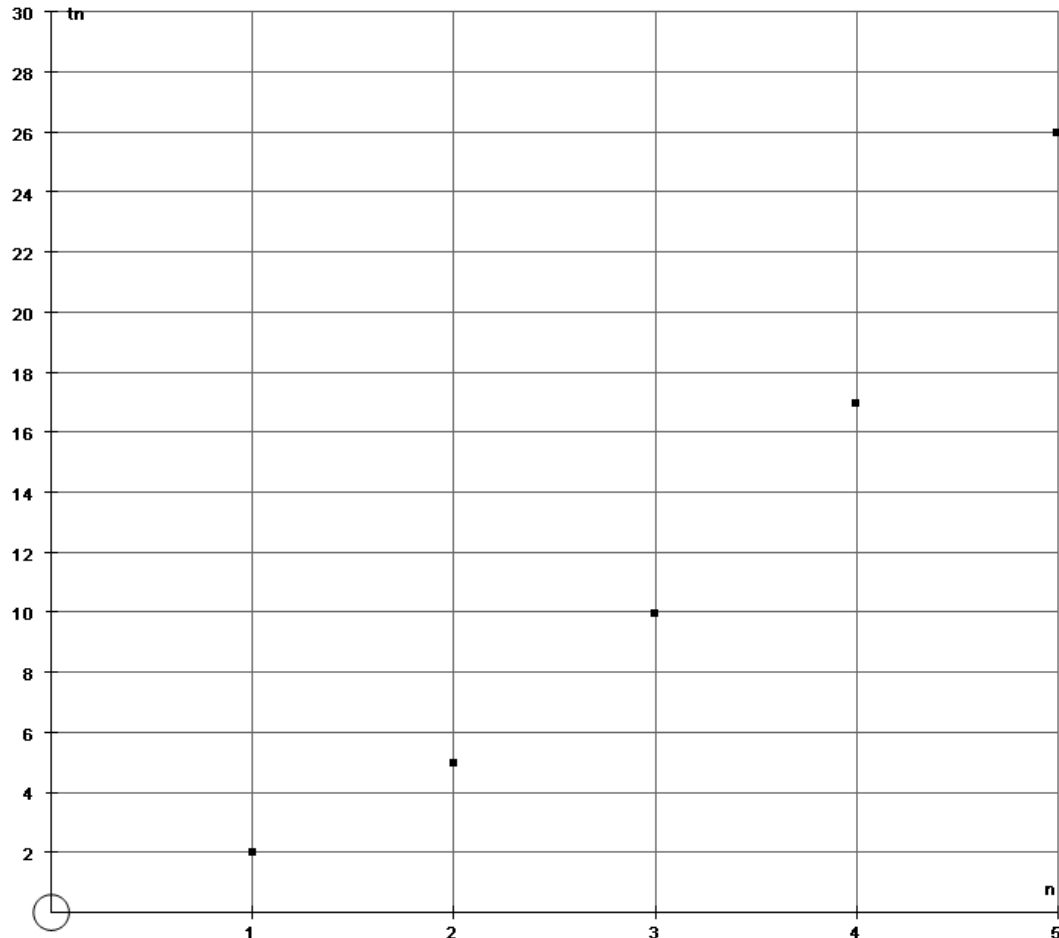
We have found very quickly that there is **NOT** a common difference. Therefore, this sequence is not arithmetic.

- The third step is to make a table of values using the pattern of dots:

Picture Number (n)	1	2	3	4	5
Number of Dots (t_n)	2	5	10	17	26

First level difference		V	V	V	V
Between terms (D_1)	→	3	5	7	9
Second level difference		V	V	V	
Between terms (D_2)	→	2	2	2	

- By looking at the values in the first level difference between successive terms (D_1), we could conclude that the sequence is not arithmetic because the values do not equal the same number.
- We find, however, that the values in the second level difference between successive terms (D_2) **DO** equal the same number.
- The fourth step is to plot the graph of n vs t_n .



By looking at the graph, we find that it isn't a straight line. If we were to connect the points with a smooth curve, we would find that the graph would have the shape of half of a parabola. Therefore, our sequence would be quadratic.

- The fifth step is to write the equation of the sequence that we graphed. By looking at our sequence and the graph, we can see that the equation of the sequence should be: $t_n = n^2 + 1$.

Sequence Summary

- An **arithmetic sequence** has the following properties:
 - The slope of the relation is the common difference **d**.
 - The common difference occurs at **D₁**.
 - The relation will look linear when graphed.
 - It is of the form $t_n = an + b$.
 - The sequence relation can be found by using the formula $t_n = t_1 + (n - 1)d$.
 - The graph of any sequence is discrete.
 - The value of **D₁** equals the slope.
 - The domain of any sequence is $D = \{n \mid n \in \mathbb{N}\}$ (i.e. the set of natural numbers: 1, 2, 3, 4, ...).
- A **quadratic sequence** has the following properties:
 - The common difference occurs at **D₂**.
 - The relation will look like a parabola when graphed.
 - It is of the form $t_n = an^2 + bn + c$.
 - The graph of any sequence is discrete.
 - The value of **D₂** is equal to **2a**.
 - The domain of any sequence is $D = \{n \mid n \in \mathbb{N}\}$ (i.e. the set of natural numbers: 1, 2, 3, 4, ...).

Example

- Determine if the sequence $\{-6, 5, 22, 45, 74, 109, \dots\}$ is arithmetic or quadratic.

We need to find out if there is a common difference at **D₁** or **D₂**:

		$\{-6, 5, 22, 45, 74, 109, \dots\}$
First level difference		V V V V V
Between terms (D₁)	→	11 17 23 29 35
Second level difference		V V V V
Between terms (D₂)	→	6 6 6 6

Since there is a second level common difference (**D₂**), the given sequence is quadratic.