Cubic And Quartic Sequences

- As we have seen in previous lessons, arithmetic sequences have a common difference at D_1 and quadratic sequences have a common difference at D_2. Similarly, cubic sequences ($t_n = an^3 + bn^2 + cn + d$) have a common difference at D_3 and quartic sequences ($t_n = an^4 + bn^3 + cn^2 + dn + e$) have a common difference at D_4.
- Example 1:
  Show that the sequence $\{4,\ 10,\ 24,\ 52,\ 100,\ 174,\ ...\}$ is cubic.

		$\{4,\ 10,\ 24,\ 52,\ 100,\ 174,\ ...\}$
First level difference		V V V V V
Between terms (D_1)	$\rightarrow$	6 14 28 48 74
Second level difference		V V V V
Between terms (D_2)	$\rightarrow$	8 14 20 26
Third level difference		V V V
Between terms (D_3)	$\rightarrow$	6 6 6

Since there is a common difference at D_3, the given sequence is cubic.

Summary

- An arithmetic sequence ($t_n = an + b$) has the following properties:
  - The common difference occurs at D_1.
  - The relation will look linear when graphed.
  - The sequence relation can be found using the formula $t_n = t_1 + (n-1)d$.
  - The value of D_1 equals the slope.
- A quadratic sequence ($t_n = an^2 + bn + c$) has the following properties:
  - The common difference occurs at D_2.
  - The relation will look like a parabola when graphed.
  - The value of $D_2 = 2a$.
- A cubic sequence ($t_n = an^3 + bn^2 + cn + d$) has the following properties:
  - The common difference occurs at D_3.
- A quartic sequence ($t_n = an^4 + bn^3 + cn^2 + dn + e$) has the following properties:
  - The common difference occurs at D_4.
- Note that the domain of a sequence is $D = \{n \mid n \in \mathbb{N}\}$ (i.e. the set of natural numbers: 1, 2, 3, 4, …).