

Cubic And Quartic Sequences

- As we have seen in previous lessons, arithmetic sequences have a common difference at D_1 and quadratic sequences have a common difference at D_2 . Similarly, cubic sequences ($t_n = an^3 + bn^2 + cn + d$) have a common difference at D_3 and quartic sequences ($t_n = an^4 + bn^3 + cn^2 + dn + e$) have a common difference at D_4 .
- Example 1:
Show that the sequence $\{1, 2, 9, 28, 65, 126, \dots\}$ is cubic.

	$\{1, 2, 9, 28, 65, 126, \dots\}$
First level difference	V V V V V
Between terms (D_1) →	1 7 19 37 61
Second level difference	V V V V
Between terms (D_2) →	6 12 18 24
Third level difference	V V V
Between terms (D_3) →	6 6 6

Since there is a common difference at D_3 , the given sequence is cubic.

- In all of our previous examples, the independent variable increases by increments of 1. There are situations, however, where the data may increase by something different. In these types of situations for a quadratic function, we use the formula $D_2 = 2ak^2$, where k is the increment of the independent variable.
- Example 2:
Algebraically determine the value of 'a' for the quadratic function with the given data:

x	0.0	0.2	0.4	0.6	0.8	1.0
y	5.00	5.52	5.88	6.08	6.12	6.00

Solution:

x	0.0	0.2	0.4	0.6	0.8	1.0
y	5.00	5.52	5.88	6.08	6.12	6.00

V V V V V

$$D_1 \rightarrow 0.52 \quad 0.36 \quad 0.20 \quad 0.04 \quad -0.12$$

V V V V

$$D_2 \rightarrow -0.16 \quad -0.16 \quad -0.16 \quad -0.16$$

Since there is a common difference at D_2 , the given function is quadratic. Since the independent variable increases by 0.2, then $k = 0.2$.

Therefore,

$$D_2 = 2ak^2$$
$$-0.16 = 2a(0.2)^2$$
$$-0.16 = 0.08a$$
$$a = -2$$

Summary

- An arithmetic sequence ($t_n = an + b$) has the following properties:
 - common difference occurs at D_1
 - the relation will appear linear when graphed
 - the sequence relation can be found using the formula
$$t_n = t_1 + (n - 1)d$$
 - the value of $D_1 = \text{slope}$
- A quadratic sequence ($t_n = an^2 + bn + c$) has the following properties:
 - common difference occurs at D_2
 - the relation will appear parabolic when graphed
 - the value of $D_2 = 2a$
- A cubic sequence ($t_n = an^3 + bn^2 + cn + d$) has the following properties:
 - common difference occurs at D_3
 - the value of $D_3 = 6a$
- A quartic sequence ($t_n = an^4 + bn^3 + cn^2 + dn + e$) has the following properties:
 - common difference occurs at D_4
 - the value of $D_4 = 24a$
- In general, the value of $D_n = n!a$, where $n! = n(n - 1)(n - 2)\dots(3)(2)(1)$. For example, $D_5 = 5!a = (5)(4)(3)(2)(1)a = 120a$.
- If the independent variable of a quadratic sequence increases by increments of a value k other than 1, then $D_2 = 2ak^2$.
- Note that the domain of a sequence is $D = \{n \mid n \in \mathbb{N}\}$ (i.e. the set of natural numbers: 1, 2, 3, 4, ...).