

Creating Quadratic Functions

- In our previous lessons, we have studied sequences where the equation was given to us. We will now examine problems in which we can determine the equation of a quadratic sequence by solving a system of equations. In the grade 11 course, we found that we could use three points to find the equation of a quadratic function of the form $y = ax^2 + bx + c$. We needed to use three points because there were three variables. Now, we are working with a quadratic sequence of the form $t_n = an^2 + bn + c$. Using the fact that $D_2 = 2a$, we can determine the value of the variable a by calculating the value of D_2 and rearranging the formula to be $a = \frac{D_2}{2}$. Because of this, we only need two points to determine the values of the variables b and c .
- Example:

(a.) Create a quadratic function $t_n = an^2 + bn + c$ to generate the sequence $\{1, 3, 6, 10, 15, 21, \dots\}$.

(b.) Use the function to determine the 20th term in the sequence.

Solution:

(a.)

Step 1: Determine the value of D_2 .

		$\{1, 3, 6, 10, 15, 21, \dots\}$
First level difference		V V V V V
Between terms (D_1)	→	2 3 4 5 6
Second level difference		V V V V
Between terms (D_2)	→	1 1 1 1

Since there is a common difference at D_2 , the given sequence is quadratic where $D_2 = 1$.

So, $a = \frac{D_2}{2}$
 $a = \frac{1}{2}$

Therefore, we have the quadratic function $t_n = \frac{1}{2}n^2 + bn + c$.

Step 2: Select any two values from the sequence and substitute into the quadratic function. This is possibly more easily done if we look at the sequence as a table of values:

n	1	2	3	4	5	6
t_n	1	3	6	10	15	21

Therefore, we can use any two points from the table above. We will use the points (1, 1) and (2, 3), where $t_1 = 1$ and $t_2 = 3$.

Step 3: Substitute the points (1, 1) and (2, 3) into the quadratic function $t_n = \frac{1}{2}n^2 + bn + c$ to create a system of two quadratic equations.

$$t_n = \frac{1}{2}n^2 + bn + c$$

$$t_1 = \frac{1}{2}(1)^2 + b(1) + c$$

$$1 = \frac{1}{2} + b + c$$

$$b + c = 1 - \frac{1}{2}$$

$$b + c = \frac{1}{2}$$

$$t_n = \frac{1}{2}n^2 + bn + c$$

$$t_2 = \frac{1}{2}(2)^2 + b(2) + c$$

$$3 = \frac{4}{2} + 2b + c$$

$$2b + c = 3 - 2$$

$$2b + c = 1$$

We now have the system of equations: $\begin{cases} b + c = \frac{1}{2} \\ 2b + c = 1 \end{cases}$.

Step 4: Solve the system of equations to determine the values of b and c .

Using substitution, we can solve the system of equations in the following way:

Rearranging Equation 1, we get:

$$b + c = \frac{1}{2}$$

$$c = \frac{1}{2} - b$$

Substituting $c = \frac{1}{2} - b$ into Equation 2, we get:

$$2b + \left(\frac{1}{2} - b\right) = 1$$

$$b + \frac{1}{2} = 1$$

$$b = 1 - \frac{1}{2}$$

$$b = \frac{1}{2}$$

Substituting $b = \frac{1}{2}$ into Equation 1, we get:

$$\frac{1}{2} + c = \frac{1}{2}$$

$$c = 0$$

Therefore, the solution to the system of equations is: $b = \frac{1}{2}$ and $c = 0$.

Step 5: Substitute the values of b and c into the quadratic

function $t_n = \frac{1}{2}n^2 + bn + c$.

Replacing the values of b and c, we now have the quadratic

function $t_n = \frac{1}{2}n^2 + \frac{1}{2}n$.

$$(b.) \quad t_n = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$t_{20} = \frac{1}{2}(20)^2 + \frac{1}{2}(20)$$

$$t_{20} = \frac{1}{2}(400) + 10$$

$$t_{20} = 210$$

Therefore, the 20th term of the sequence is 210.