

Finite Differences

- In our previous lesson, we used the substitution method to determine the equation ($t_n = an^2 + bn + c$) for a quadratic sequence. We can also use a table of finite differences to determine the equation.
- The table of finite differences for a quadratic equation is given below:

Value of n	Formula ($t_n = an^2 + bn + c$)	D_1	D_2
1	$a + b + c$		
2	$4a + 2b + c$	$3a + b$	
3	$9a + 3b + c$	$5a + b$	$2a$
4	$16a + 4b + c$	$7a + b$	$2a$
5	$25a + 5b + c$	$9a + b$	$2a$

We use this table in order to find the values of “a”, “b” and “c” for the equation $t_n = an^2 + bn + c$ that describes the quadratic sequence that we are investigating.

- Before we can use finite differences, we need to know what type of sequence we have. Each type of sequence (arithmetic, quadratic, cubic, etc.) has a different finite differences table, so we need to find at what level the common difference occurs.
- Example:
Determine the equation that generates the n^{th} term of the sequence $\{0, 10, 26, 48, 76, 110, \dots\}$.

Step 1: Find the level where the common difference occurs.

		$\{0, 10, 26, 48, 76, 110, \dots\}$
First level difference		V V V V V
Between terms (D_1)	→	10 16 22 28 34
Second level difference		V V V V
Between terms (D_2)	→	6 6 6 6

Since there is a common difference at D_2 , the given sequence is quadratic with $D_2 = 6$.

Step 2: Use the table of finite differences for the type of sequence that we have to calculate the values for the variables to make the appropriate equation.

In this situation, we have a quadratic sequence, so we will use the table of finite differences for a quadratic equation.

Value of n	Formula ($t_n = an^2 + bn + c$)	D_1	D_2
1	$a + b + c$		
2	$4a + 2b + c$	$3a + b$	$2a$
3	$9a + 3b + c$	$5a + b$	$2a$
4	$16a + 4b + c$	$7a + b$	$2a$
5	$25a + 5b + c$	$9a + b$	

The selected entry (in the box) from the D_2 column tells us that for this sequence:

$$2a = 6$$

$$a = \frac{6}{2}$$

$$a = 3$$

The selected entry (in the box) from the D_1 column tells us that for this sequence:

$$5a + b = 16$$

$$5(3) + b = 16$$

$$15 + b = 16$$

$$b = 1$$

The selected entry (in the box) from the **Formula** column tells us that for this sequence:

$$a + b + c = 0$$

$$3 + 1 + c = 0$$

$$4 + c = 0$$

$$c = -4$$

Step 3: Use the values of the variables that we have calculated to write the equation representing the sequence.

Using the values of “a”, “b” and “c” that we have calculated for the quadratic sequence, we now have:

$$t_n = an^2 + bn + c$$

$$t_n = 3n^2 + n - 4$$

Therefore, the n^{th} term of the sequence is given by the equation $t_n = 3n^2 + n - 4$.