

# Vertical Stretch

## Vertical Stretch

$$\frac{1}{a}y = x^2$$

- Another transformation that can be made to the base function  $y = x^2$  is to change the **vertical stretch**. The vertical stretch (**VS**) affects the graph by **stretching (or compressing) the graph vertically**. This will give us an image graph that looks **wider (or narrower) than the original base graph**.
- Example 1:  
Let's look at the following functions. We will use a mapping rule to generate a table of values for the image function and then graph all three functions on the same axes.

(a.)  $y = x^2$

(b.)  $\frac{1}{3}y = x^2$

(c.)  $3y = x^2$

$(x, y) \rightarrow (x, 3y)$

$(x, y) \rightarrow (x, \frac{1}{3}y)$

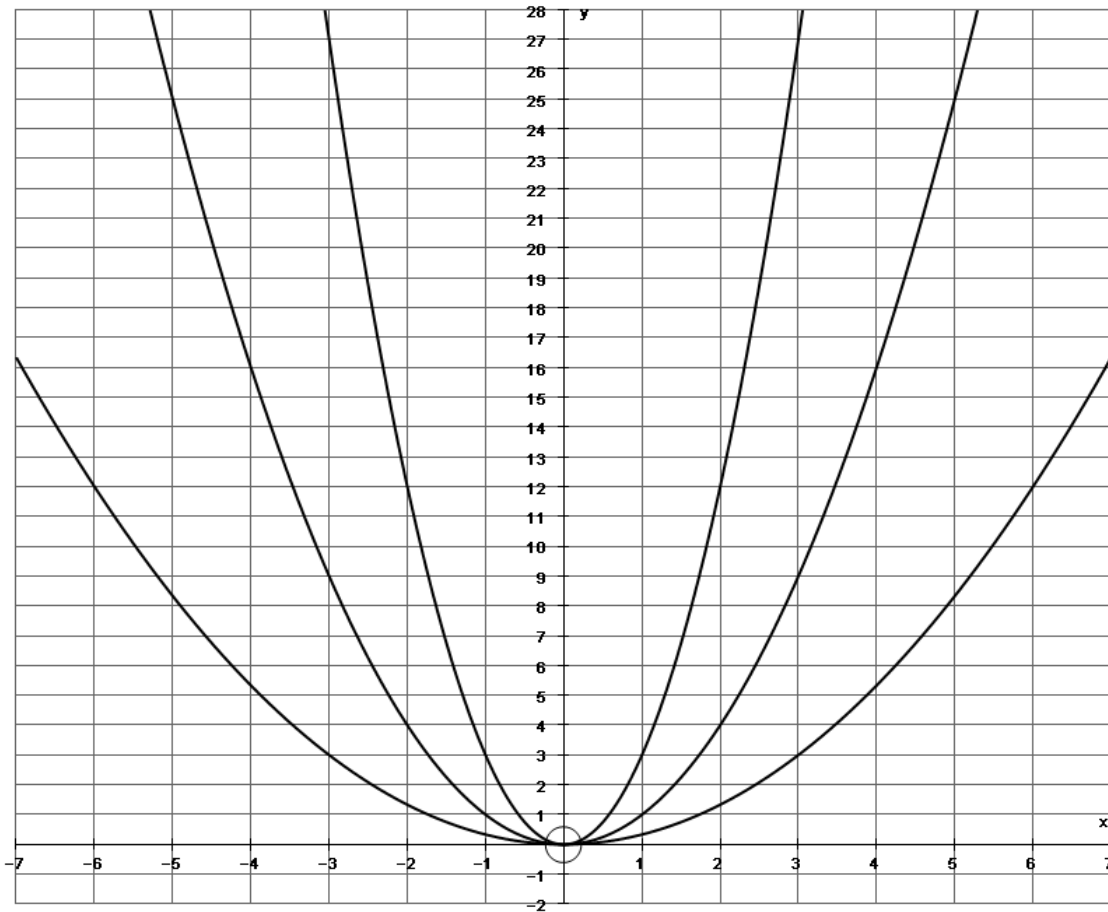
| x  | y |
|----|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0  | 0 |
| 1  | 1 |
| 2  | 4 |
| 3  | 9 |

| x  | y  |
|----|----|
| -3 | 27 |
| -2 | 12 |
| -1 | 3  |
| 0  | 0  |
| 1  | 3  |
| 2  | 12 |
| 3  | 27 |

| x  | y             |
|----|---------------|
| -3 | $\frac{4}{3}$ |
| -2 | $\frac{1}{3}$ |
| -1 | $\frac{1}{3}$ |
| 0  | 0             |
| 1  | $\frac{1}{3}$ |
| 2  | $\frac{4}{3}$ |
| 3  | 3             |

multiplied by 3

multiplied by  $\frac{1}{3}$



- When there is a  $\frac{1}{3}y$  in the function, then there is a vertical stretch of 3. This will result in a graph that is **narrower** than the graph of  $y = x^2$ .
- When there is a  $3y$  in the function, then there is a vertical stretch of  $\frac{1}{3}$ . This will result in a graph that is **wider** than the graph of  $y = x^2$ .
- The vertical stretch factor is the **reciprocal** of the coefficient of  $y$  in the function.

- To write the mapping rule of the function, look at what happens to the numerical coefficient of  $y$ :

| <u>Equation</u>      | <u>Mapping Rule</u>                    |
|----------------------|--|
| $\frac{1}{3}y = x^2$ | $(x, y) \rightarrow (x, 3y)$           |
| VS = 3               |  |
| $3y = x^2$           | $(x, y) \rightarrow (x, \frac{1}{3}y)$ |
| VS = $\frac{1}{3}$   |  |

- In the equation, the Vertical Stretch factor (VS) is the **reciprocal** of the numerical coefficient of  $y$ . However, in the mapping rule, the VS is the **same** as the numerical coefficient of  $y$ .
- Example 2:  
Find the mapping rules for the following functions:

(a.)  $6y = x^2$       (b.)  $\frac{1}{2}y = x^2$

- (a.) In the function  $6y = x^2$ , the mapping rule will be  $(x, y) \rightarrow (x, \frac{1}{6}y)$ , where the VS =  $\frac{1}{6}$ .
- (b.) In the function  $\frac{1}{2}y = x^2$ , the mapping rule will be  $(x, y) \rightarrow (x, 2y)$ , where the VS = 2.

## Summary

Equation:

$$\frac{1}{a}y = x^2$$

$$\boxed{VS = a}$$

Mapping Rule:

$$(x, y) \rightarrow (x, ay)$$

- The Vertical Stretch is the **RECIPROCAL** of the value multiplied by y in the equation.
- The Vertical Stretch is the **SAME** as the value multiplied by y in the mapping rule.
- When  $VS > 1$ , the graph is **NARROWER** in appearance.
- When  $VS < 1$ , the graph is **WIDER** in appearance.