

General Form Of A Quadratic Function

- We have seen that Transformational Form has many advantages. It allows us to graph a function and identify key components from the equation itself, such as the vertex (h, k) , and the axis of symmetry $(x = h)$. There is another form known as the **General Form** of a function:

GENERAL FORM

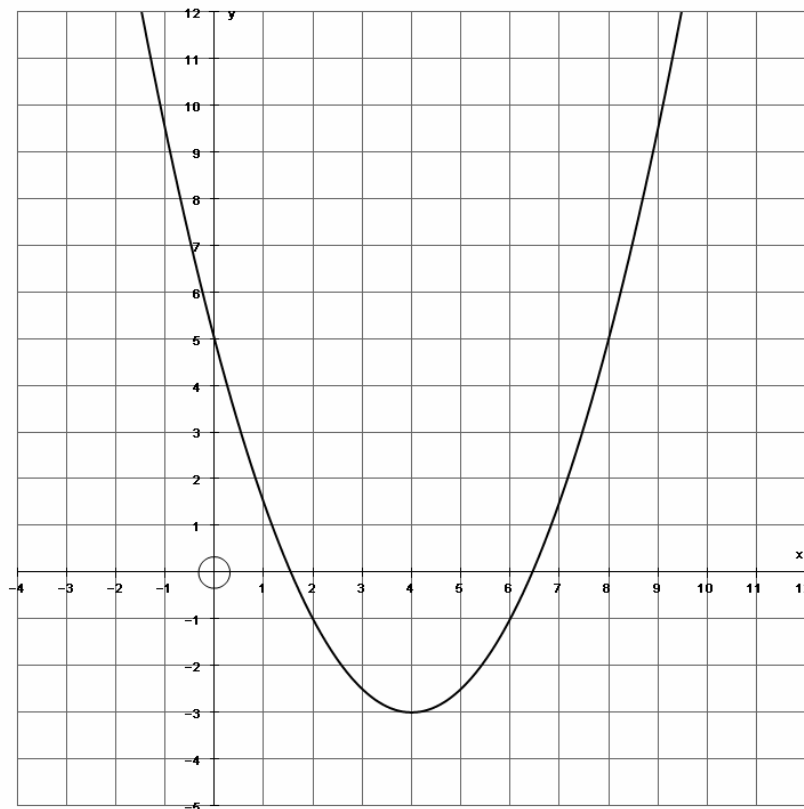
$$y = ax^2 + bx + c, \text{ where } a \neq 0$$

- To see the benefits of expressing a quadratic function in general form, let us examine the following graph of

$$y = \frac{1}{2}x^2 - 4x + 5.$$

- Example 1:

Use the graph of the function $y = \frac{1}{2}x^2 - 4x + 5$ to answer the following questions.



(a.) Identify the values of 'a', 'b', and 'c'.

$$a = \frac{1}{2} \quad b = -4 \quad c = 5$$

(b.) Use the graph to identify the vertex.

The vertex is (4, -3).

(c.) Use the graph to identify the y-intercept.

The y-intercept is (0, 5).

(d.) Evaluate the y-intercept algebraically (let $x = 0$).

$$y = \frac{1}{2}x^2 - 4x + 5$$

$$y = \frac{1}{2}(0)^2 - 4(0) + 5$$

$$y = 0 - 0 + 5$$

$$y = 5$$

The y-intercept is (0, 5).

(e.) Identify the Vertical Stretch factor.

$$VS = \frac{1}{2}$$

(f.) What effect will 'a' have on the graph if $a > 0$ or $a < 0$.

If $a > 0$, the graph opens upwards.

If $a < 0$, the graph opens downwards.

(g.) Determine the value of $x = -\frac{b}{2a}$.

$$x = -\frac{b}{2a} = -\frac{(-4)}{2\left(\frac{1}{2}\right)} = \frac{4}{1}$$

Therefore, $x = 4$.

(h.) What does this value indicate on the graph?

$-\frac{b}{2a}$ is the x-coordinate of the vertex and the numerical value for the axis of symmetry.

(i.) State the Domain and Range.

$$\text{Domain: } D = \{x \mid x \in \mathbb{R}\}$$

$$\text{Range: } R = \{y \mid y \geq -3, y \in \mathbb{R}\}$$

- Example 2:

Use the General Form of the function $y = 3x^2 + 12x + 17$ to answer the following questions.

(a.) Determine the y-intercept.

(b.) Use the equation $x = -\frac{b}{2a}$ to identify the axis of symmetry.

(c.) Determine the coordinates of the vertex.

Solution:

(a.) To find the y-intercept, plug $x = 0$ into the General Form $y = 3x^2 + 12x + 17$.

$$\begin{aligned}y &= 3x^2 + 12x + 17 \\y &= 3(0)^2 + 12(0) + 17 \\y &= 17\end{aligned}$$

(b.) In the General Form, $y = 3x^2 + 12x + 17$, $a = 3$ and $b = 12$:

$$x = -\frac{b}{2a} = -\frac{12}{2(3)} = -\frac{12}{6} = -2$$

Therefore, the axis of symmetry is $x = -2$.

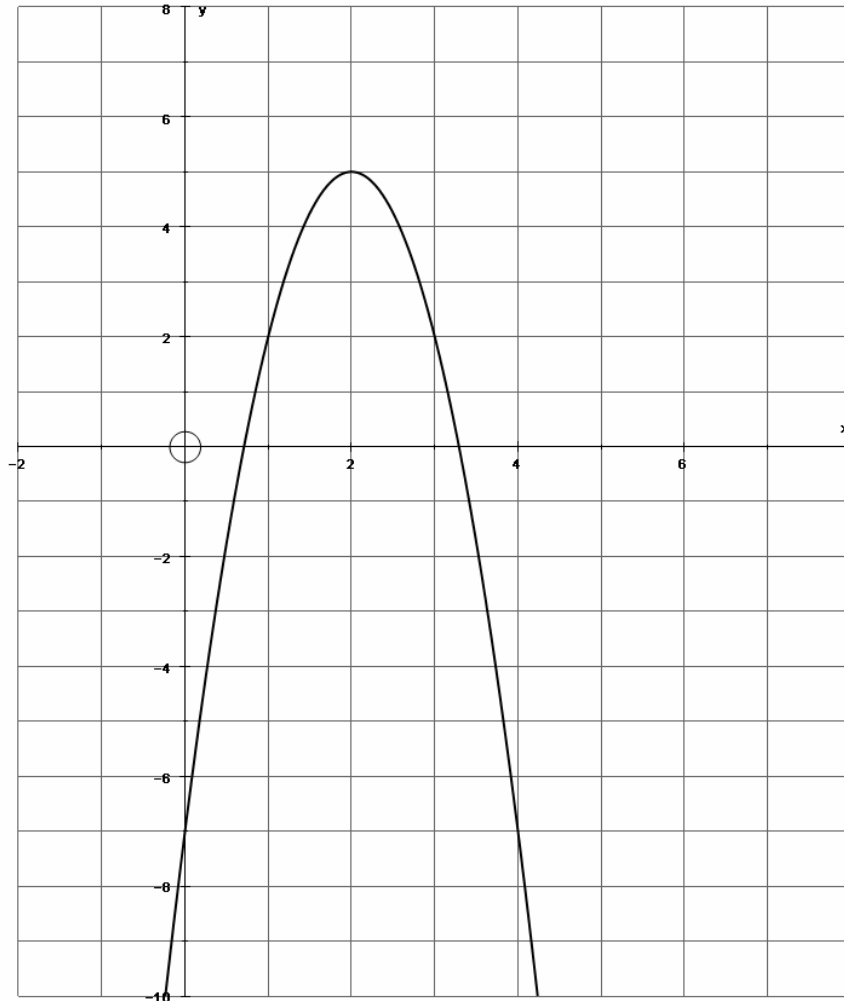
(c.) To identify the vertex, the axis of symmetry is the x-coordinate of the vertex. Therefore, to find the y-coordinate, we substitute the value of $x = -2$ back into the original equation.

$$\begin{aligned}y &= 3x^2 + 12x + 17 \\y &= 3(-2)^2 + 12(-2) + 17 \\y &= 12 - 24 + 17 \\y &= 5\end{aligned}$$

Therefore, the vertex is located at $(-2, 5)$.

- Example 3:

Use the graph of the function $y = -3x^2 + 12x - 7$ to answer the following questions.



(a.) Use the graph to identify the axis of symmetry.
The axis of symmetry is $x = 2$.

(b.) Use $x = -\frac{b}{2a}$ to get the axis from the equation.

$$x = -\frac{b}{2a} = -\frac{12}{2(-3)} = -\frac{12}{(-6)} = 2$$

Therefore, the axis of symmetry is $x = 2$.

(c.) Use the graph to identify the vertex.

The vertex is $(2, 5)$.

(d.) How could you get the vertex from the equation?

We can solve for the x-value of the vertex by using

$x = -\frac{b}{2a}$. We can then plug that x-value into the equation

and solve for the value of y. The value of y that we find is the y-value for the vertex.

(e.) Use the graph to identify the y-intercept.

The y-intercept is (0, -7).

(f.) How could you get the y-intercept from the equation?

When an equation is in General Form ($y = ax^2 + bx + c$), the y-intercept is the value of the variable c.

(g.) Use the graph to identify the vertical stretch.

We can use the "slope" method. If we go over 1 horizontally along the x-axis, we would then go down 3 vertically until we intersected the graph. In our base $y = x^2$ graph, we would go over 1 horizontally and then go up 1 vertically until we intersected the graph. We have now tripled the vertical distance that we need to move, so our vertical stretch must be 3.

(h.) How could you get the vertical stretch from the equation?

When using General Form, where our equation is of the form $y = ax^2 + bx + c$, the vertical stretch is the absolute value of the variable a.

Summary

General Form: $y = ax^2 + bx + c$, where $a \neq 0$

- $a > 0$ will produce a parabola that opens upward
- $a < 0$ will produce a parabola that opens downward
- the y-intercept is $y = c$
- the axis of symmetry is $x = -\frac{b}{2a}$, which is also the x-coordinate of the vertex
- the Range of the parabola is $\{y \mid y \geq 0\}$ if $a > 0$, and $\{y \mid y \leq 0\}$ if $a < 0$
- the vertical stretch factor is $|a|$