

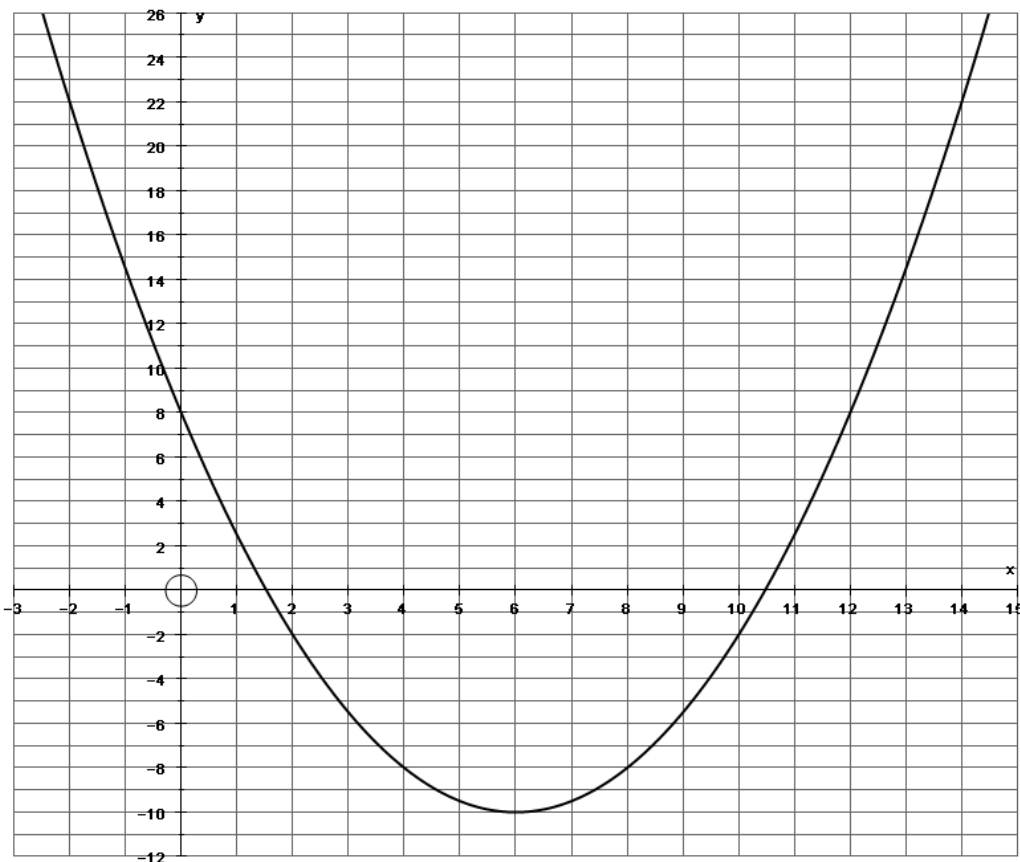
## General Form Of A Quadratic Function

- We have seen that Transformational Form has many advantages. It allows us to graph a function and identify key components from the equation itself, such as the vertex  $(h, k)$ , and the axis of symmetry  $(x = h)$ . There is another form of a function known as **General Form**:

### GENERAL FORM

$$y = ax^2 + bx + c, \text{ where } a \neq 0$$

- There are also advantages when we express a quadratic function in General Form. To observe some of them, we are going to look at the graph of  $y = \frac{1}{2}x^2 - 6x + 8$ .
- Example 1:  
Use the graph of the function  $y = \frac{1}{2}x^2 - 6x + 8$  to answer the following questions.



(a.) Identify the values of “a”, “b”, and “c”.

$$a = \frac{1}{2} \quad b = -6 \quad c = 8$$

(b.) Use the graph to identify the vertex.

The vertex is (6, -10).

(c.) Use the graph to identify the y-intercept.

The y-intercept is (0, 8).

(d.) Evaluate the y-intercept algebraically (let  $x = 0$ ).

$$y = \frac{1}{2}x^2 - 6x + 8$$

$$y = \frac{1}{2}(0)^2 - 6(0) + 8$$

$$y = 0 - 0 + 8$$

$$y = 8$$

The y-intercept is (0, 8).

(e.) Identify the Vertical Stretch factor.

$$VS = \frac{1}{2}$$

(f.) What effect will “a” have on the graph if  $a > 0$  or  $a < 0$ .

If  $a > 0$ , the graph opens upwards.

If  $a < 0$ , the graph opens downwards.

(g.) Determine the value of  $x = -\frac{b}{2a}$ .

$$x = -\frac{b}{2a} = -\frac{(-6)}{2\left(\frac{1}{2}\right)} = \frac{6}{1}$$

Therefore,  $x = 6$ .

(h.) What does this value indicate on the graph?

$-\frac{b}{2a}$  is the x-coordinate of the vertex and the numerical value for the axis of symmetry.

(i.) State the Domain and Range.

$$\text{Domain: } D = \{x \mid x \in \mathbb{R}\}$$

$$\text{Range: } R = \{y \mid y \geq -10, y \in \mathbb{R}\}$$

- Example 2:

Use the General Form of the function  $y = 4x^2 - 24x + 16$  to answer the following questions.

(a.) Determine the y-intercept.

(b.) Use the equation  $x = -\frac{b}{2a}$  to identify the axis of symmetry.

(c.) Determine the coordinates of the vertex.

Solution:

(a.) To find the y-intercept, plug  $x = 0$  into the General Form  $y = 4x^2 - 24x + 16$ .

$$\begin{aligned}y &= 4x^2 - 24x + 16 \\y &= 4(0)^2 - 24(0) + 16 \\y &= 16\end{aligned}$$

Therefore, the y-intercept is  $(0, 16)$ .

(b.) In the General Form equation  $y = 4x^2 - 24x + 16$ ,  $a = 4$  and  $b = -24$ :

$$x = -\frac{b}{2a} = -\frac{(-24)}{2(4)} = \frac{24}{8} = 3$$

Therefore, the axis of symmetry is  $x = 3$ .

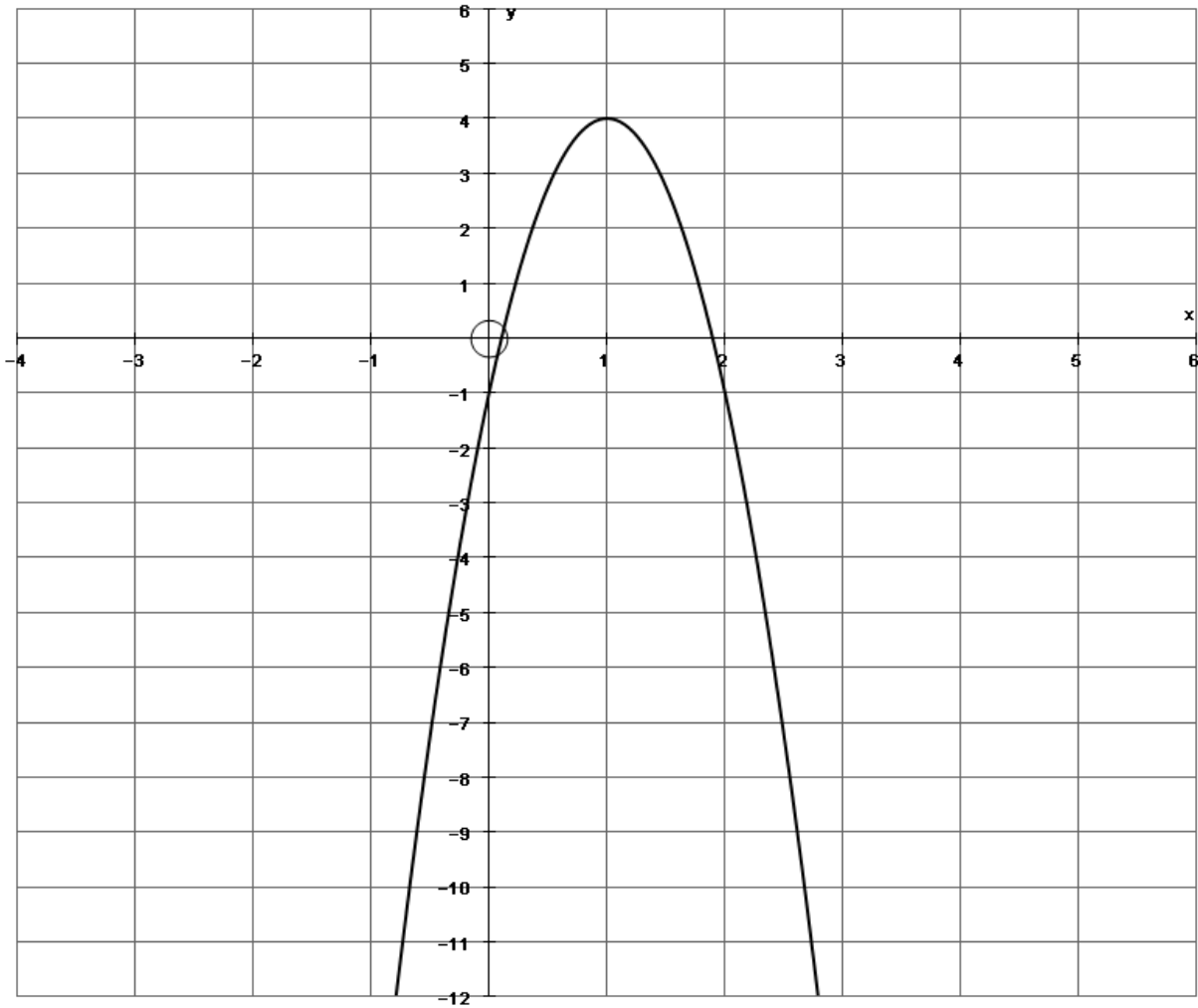
(c.) To identify the vertex, the axis of symmetry is the x-coordinate of the vertex. Therefore, to find the y-coordinate, we substitute the value of  $x = 3$  back into the original equation.

$$\begin{aligned}y &= 4x^2 - 24x + 16 \\y &= 4(3)^2 - 24(3) + 16 \\y &= 36 - 72 + 16 \\y &= -20\end{aligned}$$

Therefore, the vertex is located at  $(3, -20)$ .

- Example 3:

Use the graph of the function  $y = -5x^2 + 10x - 1$  to answer the following questions.



(a.) Use the graph to identify the axis of symmetry.

The axis of symmetry is  $x = 1$ .

(b.) Use  $x = -\frac{b}{2a}$  to get the axis from the equation.

$$x = -\frac{b}{2a} = -\frac{10}{2(-5)} = -\frac{10}{(-10)} = 1$$

Therefore, the axis of symmetry is  $x = 1$ .

(c.) Use the graph to identify the vertex.

The vertex is  $(1, 4)$ .

- (d.) How could you get the vertex from the equation?  
 We can solve for the x-value of the vertex by using  $x = -\frac{b}{2a}$ . We can then plug that x-value into the equation and solve for the value of y. The value of y that we find is the y-value for the vertex.
- (e.) Use the graph to identify the y-intercept.  
 The y-intercept is (0, -1).
- (f.) How could you get the y-intercept from the equation?  
 When an equation is in General Form ( $y = ax^2 + bx + c$ ), the y-intercept is the value of the variable c.
- (g.) Use the graph to identify the vertical stretch.  
 We can use the “slope” method. If we go over 1 horizontally along the x-axis, we would then go down 5 vertically until we intersected the graph. In our base  $y = x^2$  graph, we would go over 1 horizontally and then go up 1 vertically until we intersected the graph. We have now increased the vertical distance that we need to move by a factor of 5, so our vertical stretch must be 5.
- (h.) How could you get the vertical stretch from the equation?  
 When using General Form, where our equation is of the form  $y = ax^2 + bx + c$ , the vertical stretch is the absolute value of the variable a.

## Summary

General Form:  $y = ax^2 + bx + c$ , where  $a \neq 0$

- $a > 0$  will produce a parabola that opens upward.
- $a < 0$  will produce a parabola that opens downward.
- The y-intercept is found at  $y = c$ . (As a point, it is (0, c).)
- The axis of symmetry is  $x = -\frac{b}{2a}$ , which is also the x-coordinate of the vertex.
- The Range of the parabola is  $\{y \mid y \geq 0\}$  if  $a > 0$ , and  $\{y \mid y \leq 0\}$  if  $a < 0$ .
- The vertical stretch factor is  $|a|$ .