Standard Form Of A Quadratic Function

- We have seen that Transformational Form has many advantages. It allows us to graph a function and identify key components from the equation itself, such as the vertex $(h,\ k)$, and the axis of symmetry $(x = h)$. There is another form known as the **Standard Form** of a function:

STANDARD FORM

$$\boxed{y = a(x-h)^2 + k,\ \text{where } a \neq 0}$$

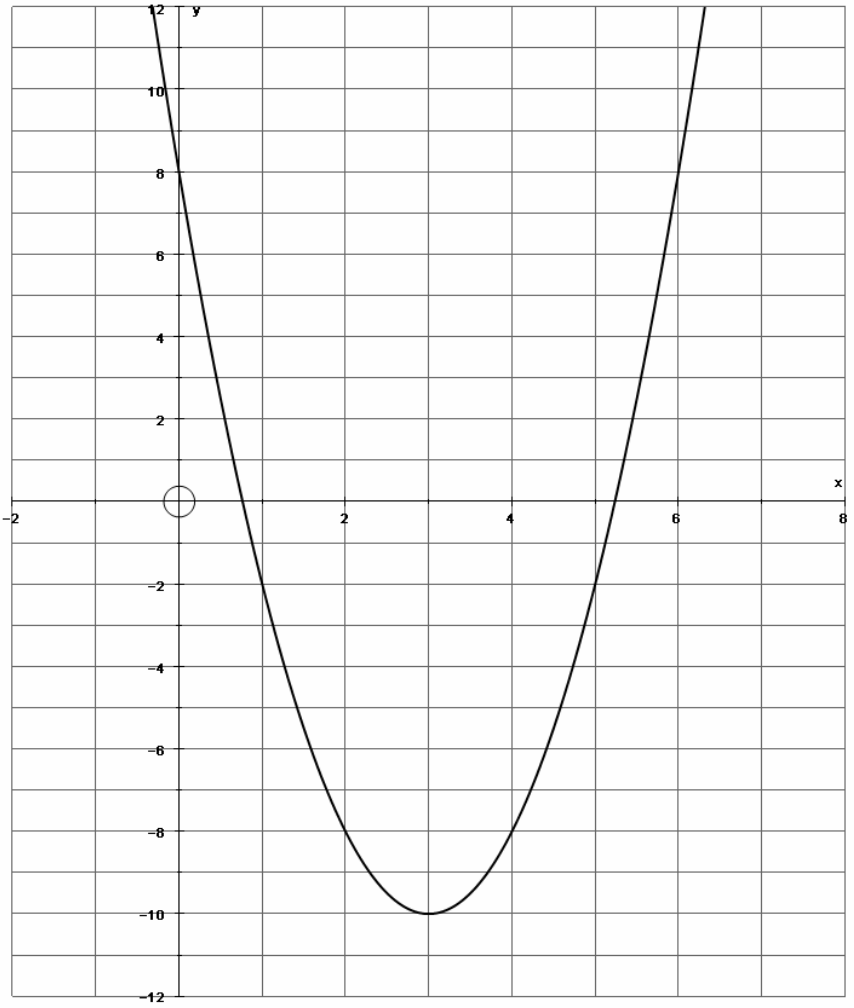
- Like Transformational Form (TF), the vertex is $(h,\ k)$ and the axis of symmetry is $x = h$. Basically, we can get the Standard Form (SF) from the Transformational Form by solving for the variable y.
- Example 1:

  Identify the axis of symmetry and vertex of $\frac{1}{2}(y+3) = (x-1)^2$.

  Solution:
  The axis of symmetry is of the form $x = h$. Therefore, the axis of symmetry is $x = 1$.

  Since the Standard Form of a function $y = a(x-h)^2 + k$ has the vertex located at $(h,\ k)$, then the function $\frac{1}{2}(y+3) = (x-1)^2$ has its vertex located at $(1,\ -3)$.

- Example 2:
  Use the graph of the function $y = 2(x-3)^2 - 10$ to answer the following questions.



(a.) Use the graph to identify the vertex.
The vertex is $(3, -10)$.

(b.) How could you get the vertex from the equation?
The x-value of the vertex is the same as the value of h in the equation and the y-value of the vertex is the same as the value of k in the equation.

(c.) Use the graph to identify the axis of symmetry.
The axis of symmetry is $x = 3$.

(d.) How could you get the axis of symmetry from the equation?
The x-value for the axis of symmetry is the same as the value of h in the equation.

(e.) Use the graph to identify the y-intercept.

The y-intercept is $(0,\ 8)$.

(f.) How could you get the y-intercept from the equation?

We could get the y-intercept from the equation by letting $x = 0$ and then solving for the value of y.

(g.) Use the graph to identify the vertical stretch.

We can use the "slope" method. If we go over 1 horizontally along the x-axis, we would then go up 2 vertically until we intersected the graph. In our base $y = x^2$ graph, we would go over 1 horizontally and then go up 1 vertically until we intersected the graph. We have now doubled the vertical distance that we need to move, so our vertical stretch must be 2.

(h.) How could you get the vertical stretch from the equation?

When using Standard Form, where our equation is of the form $y = a(x - h)^2 + k$, the vertical stretch is the absolute value of the variable a.

Summary

Standard Form: $y = a(x - h)^2 + k,\ \text{where } a \neq 0$

- $a > 0$ will produce a parabola that opens upward
- $a < 0$ will produce a parabola that opens downward
- the axis of symmetry is $x = h$
- the vertex is $(h,\ k)$
- the y-intercept is found by letting $x = 0$ and solving for y
- the Range of the parabola is $\{y \mid y \geq k\}$ if $a > 0$, and $\{y \mid y \leq k\}$ if $a < 0$
- the vertical stretch factor is $|a|$