Completing The Square

- In our last lesson, we looked at converting an equation from Standard Form into Transformational Form or General Form and Transformational Form into Standard Form or General Form. In this lesson, we will look at how to convert an equation from General Form into Transformational Form or Standard Form. To do this, we need to use a method called "completing the square". First, let's do a quick review of factoring perfect square trinomials of the form $x^2 + bx + c$.
- When factoring trinomials of this type, we try to find two numbers that multiply to give "c" and add to give "b". By listing the possible products to give "c", we look at the various combinations to find the pair that adds up to "b".
- Example 1:
  To factor $x^2 + 8x + 16$, list the different ways to multiply to get 16:

  Product is 16 $= \begin{cases} 1 \mid 16 \\ 2 \mid 8 \\ 4 \mid 4 \end{cases}$.

  Since $4 + 4 = 8$, we know that $x^2 + 8x + 16 = (x + 4)(x + 4)$.
- Example 2:
  Factor each perfect square trinomial.
  (a.) $x^2 + 10x + 25$ (b.) $x^2 - 16x + 64$ (c.) $x^2 + 20x + 100$

  (a.) Product is 25

  $\begin{cases} 1 \mid 25 \\ 5 \mid 5 \end{cases} = \text{sum}$

  Therefore, we have $(x + 5)(x + 5)$.

  (b.) Product is 64

  $\begin{cases} -1 \mid -64 \\ -2 \mid -32 \\ -4 \mid -16 \\ -8 \mid -8 \end{cases} = \text{sum}$

  Therefore, we have $(x - 8)(x - 8)$.

(c.) Product is 100

$$\left\{\begin{array}{l|l} 1 & 100 \\ 2 & 50 \\ 4 & 25 \\ 5 & 20 \\ 10 & 10 \end{array}\right. = \text{sum}$$

Therefore, we have
$(x+10)(x+10)$.

- From our examples, we can see that there is a pattern that emerges when factoring perfect square trinomials of the form x^2+bx+c:
  - To determine the value of "c", divide the value of "b" by 2 and square the result. Therefore, $c=\left(\frac{b}{2}\right)^2$.
- The process of finding a number that produces a perfect square trinomial is called "completing the square", because we are actually completing the trinomial to produce a perfect square.
- Now that we know how to find the number that we need to complete the square, we can complete our final two conversions of General Form.

<u>Converting General Form To Standard/Transformational Form</u>

- We need to complete the square to get both conversions of General Form. It is easier to convert General Form (GF) into Transformational Form (TF) and then, if we need to, convert to Standard Form (SF).
- Example 1:
  Convert the function $y=x^2+10x-7$ from General Form to Transformational Form.

  Solution:

$y=x^2+10x-7$ $\leftarrow$ GF

$y+7=x^2+10x$ $\leftarrow$ add 7 to both sides

$y+7+\left(\frac{10}{2}\right)^2=x^2+10x+\left(\frac{10}{2}\right)^2$ $\leftarrow$ add $\left(\frac{10}{2}\right)^2$ to both sides

$y + 7 + (5)^2 = x^2 + 10x + (5)^2$ ← simplify $\frac{10}{2} = 5$

$y + 7 + 25 = x^2 + 10x + 25$ ← simplify $(5)^2 = 25$

$y + 32 = (x + 5)^2$ ← TF

If the question had asked us to convert to Standard Form, we would have:

$y = (x + 5)^2 - 32$ ← SF

- Example 2:
  Convert the function $y = 2x^2 + 12x + 23$ from General Form to Transformational Form.

  Solution:

  $y = 2x^2 + 12x + 23$ ← GF

  $y - 23 = 2x^2 + 12x$ ← subtract 23 from both sides

  $\frac{1}{2}y - \frac{23}{2} = x^2 + 6x$ ← divide both sides by 2

  $\frac{1}{2}y - \frac{23}{2} + \left(\frac{6}{2}\right)^2 = x^2 + 6x + \left(\frac{6}{2}\right)^2$ ← add $\left(\frac{6}{2}\right)^2$ to both sides

  $\frac{1}{2}y - \frac{23}{2} + (3)^2 = x^2 + 6x + (3)^2$ ← simplify $\frac{6}{2} = 3$

  $\frac{1}{2}y - \frac{23}{2} + 9 = x^2 + 6x + 9$ ← simplify $(3)^2 = 9$

  $\frac{1}{2}y - \frac{23}{2} + \frac{18}{2} = (x + 3)^2$ ← use a common denominator of 2

  $\frac{1}{2}y - \frac{5}{2} = (x + 3)^2$ ← add the fractions together

  $\frac{1}{2}(y - 5) = (x + 3)^2$ ← TF

  If the question had asked us to convert to Standard Form, we would have:

  $y = 2(x + 3)^2 + 5$ ← SF