

Exponential Equations

- As we've seen earlier in this unit, there are several important properties of exponents that can be useful in simplifying expressions. The exponent properties are listed below.

Exponent Properties

A. Negative Exponents:

$$b^{-x} = \frac{1}{b^x} \text{ and } \frac{1}{b^{-x}} = b^x$$

B. Zero Exponent:

$$b^0 = 1 \text{ (for } b \neq 0\text{)}$$

C. Multiplication Rule:

$$(b^x)(b^y) = b^{x+y}$$

D. Quotient Rule:

$$\frac{b^x}{b^y} = b^{x-y}$$

E. Power Rule:

$$(b^x)^y = b^{xy}$$

F. Common Base Rule:

$$b^x = b^y \text{ if and only if } x = y$$

G. Common Exponent Rule:

$$a^x = b^x \text{ if and only if } a = b$$

- We will now look at how we can solve exponential equations by converting bases.
- Example 1:
Solve each exponential equation.

$$(a.) \quad 3^{2x} = 81$$

$$(b.) \quad 3(2^{x+4}) = 96$$

$$(c.) \quad 9^{3x-6} + 20 = 749$$

Solution

$$(a.) \quad 3^{2x} = 81$$

$$3^{2x} = 3^4$$

$$2x = 4$$

$$x = 2$$

$$(b.) \quad 3(2^{x+4}) = 96$$

$$\frac{3(2^{x+4})}{3} = \frac{96}{3}$$

$$2^{x+4} = 32$$

$$2^{x+4} = 2^5$$

$$x + 4 = 5$$

$$x = 1$$

$$(c.) \quad 9^{3x-6} + 20 = 749$$

$$9^{3x-6} = 749 - 20$$

$$9^{3x-6} = 729$$

$$(3^2)^{3x-6} = 3^6$$

$$3^{6x-12} = 3^6$$

$$6x - 12 = 6$$

$$6x = 18$$

$$x = 3$$

- We have also looked at word problems involving exponential equations in this unit. Now, let's take a second look at these types of problems and apply our new knowledge of exponent properties to solve them.
- Example 2:
Cheryl is growing a bacteria culture in a Petri dish. Initially, the population was 174 bacteria per cm^2 . If the population triples every seven hours, when will the population reach 14094 bacteria per cm^2 ?

Solution

From the information given, we know that $a = 174$, $b = 3$ and $c = 7$.

Therefore, the equation of the exponential function is $y = 174(3)^{\frac{x}{7}}$.

To determine when the population will reach 14094, we must solve the equation:

$$14094 = 174(3)^{\frac{x}{7}}$$

To solve this equation, we can first divide both sides by 174 and express each power as a common base.

$$14094 = 174(3)^{\frac{x}{7}}$$

$$\frac{14094}{174} = \frac{174(3)^{\frac{x}{7}}}{174}$$

$$81 = (3)^{\frac{x}{7}}$$

$$3^4 = (3)^{\frac{x}{7}}$$

$$4 = \frac{x}{7}$$

$$x = 28$$

Therefore, the population will reach 14094 bacteria per cm^2 in 28 hours.